Quantum Spin Chains and Ladders:

Theoretical Concepts and Recent Developments

H.-J. Mikeska

Institut für Theoretische Physik

Universität Hannover

Outline

Milestones in Low D Magnetism

1925/31	Ernst Ising, Hans Bethe (Heisenberg chain)
1944	Lars Onsager: 2D Isingmodell
1966	Mermin-Wagner theorem: strong temperature fluctuations
1971	Baxter: Eight vertex model

Milestones in Low D Magnetism

1925/31	Ernst Ising, Hans Bethe (Heisenberg chain)
1944	Lars Onsager: 2D Isingmodell
1966	Mermin-Wagner theorem: strong temperature fluctuations
1971	Baxter: Eight vertex model
1972	Neutron scattering on TMMC
1983	Haldane conjecture: strong quantum fluctuations
1986	High T_c superconductivity based on 2D AF's

Milestones in Low D Magnetism

1925/31 Ernst Ising, Hans Bethe (Heisenberg chain)

1944 Lars Onsager: 2D Isingmodell

1966 Mermin-Wagner theorem: strong temperature fluctuations

1971 Baxter: Eight vertex model

1972 Neutron scattering on TMMC

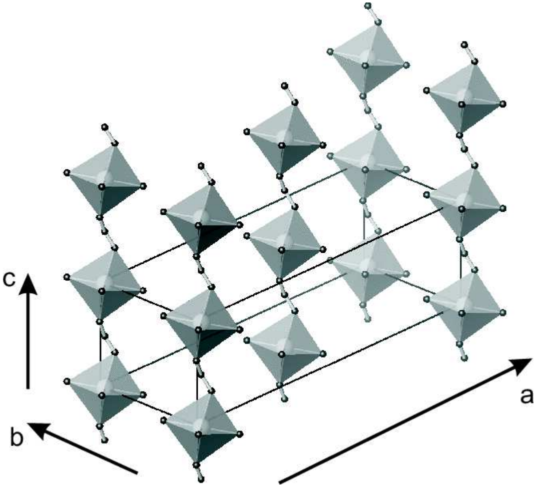
1983 Haldane conjecture: strong quantum fluctuations

1986 High T_c superconductivity based on 2D AF's

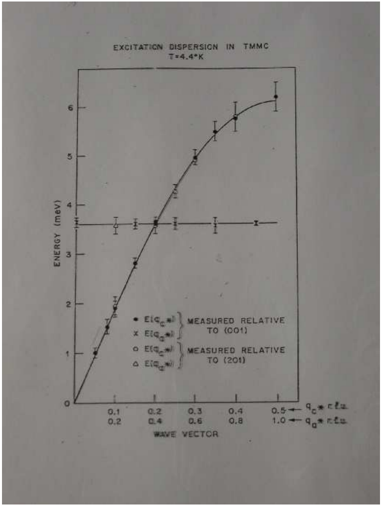
since 1990 quantum phase diagrams / magnetization plateaus
order from disorder / BEC of quantum magnets /
quantum solitons

Low-dimensional magnets exist as real crystals

example: $Ni(C_5H_{14}N_2)_2N_3(ClO_4)$ = NDMAZ



experimental check of low dimensionality



$((CH)_3)_4NMnCl_3$

= TMMC

$S = \frac{5}{2}$

inelastic neutron scattering:

Hutchings, Shirane, Birgeneau and Holt (1972)

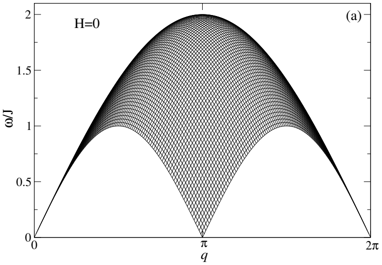
prototypes of low dimensional magnets:

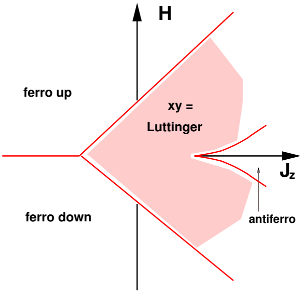
- S=1/2 Heisenberg chain
- 1D quantum spin systems with gap and rotationally invariant exchange
- $S = 1/2$ chain with orbital degree of freedom
- 2D S=1/2 Heisenberg magnets

low D prototypes (1): S=1/2 Heisenberg chain

$$\mathcal{H} = \sum_n J\left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y\right) + J_z S_n^z S_{n+1}^z - H \sum_n S_n^z$$

$-J < J_z < +J$: gapless algebraic spin liquid / excitation continuum
interacting fermions
with non Fermi (Luttinger) liquid behaviour





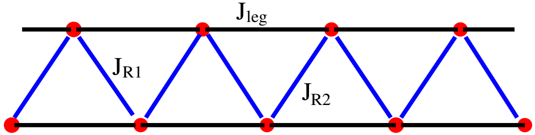
prototypes of low dimensional magnets:

- S=1/2 Heisenberg chain
- 1D quantum spin systems with gap and rotationally invariant exchange
- $S = 1/2$ chain with orbital degree of freedom
- 2D S=1/2 Heisenberg magnets

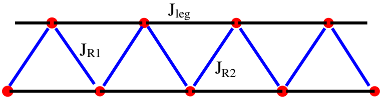
low D prototypes (2): 1D quantum spin systems with gap and rotationally invariant exchange

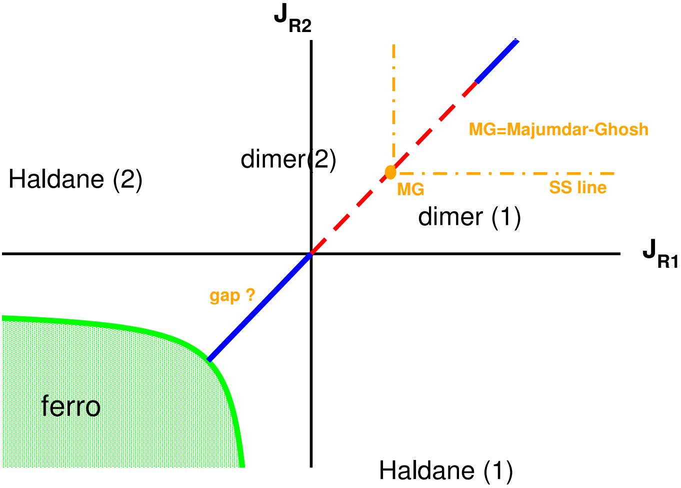
- $S = 1$ Heisenberg antiferromagnet: Haldane gap $\Delta \approx 0.41..J$
- $S = 1/2$ HAF with NN exchange J and NNN exchange J_2:
  $J_2 > 0.2411...J$: excitation gap, 2 degenerate ground states
  $J_2 = 0.5J$: Majumdar-Ghosh chain (exact dimer ground states)
  $0.5J < J_2 < 1.25J$: magnetization plateau at 1/3 saturation
- $S = 1/2$ two leg ladder: excitation gap $\Delta \approx 0.5J$

unified view: $S = 1/2$ zig-zag ladder



zig-zag ladder





prototypes of low dimensional magnets:

- S=1/2 Heisenberg chain
- 1D quantum spin systems with gap and rotationally invariant exchange: dimer aspects
- $S = 1/2$ chain with orbital degree of freedom
- 2D S=1/2 Heisenberg magnets

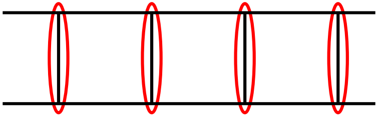
dimer aspects of gapped spin systems

$S = \frac{1}{2}$ dimer: $\mathcal{H} = J\,\mathbf{S}_1 \cdot \mathbf{S}_2 \qquad \Longrightarrow \; \Delta E = J$

interacting $S = \frac{1}{2}$ dimers: $\mathcal{H} = J \sum_{\vec{n}} \mathbf{S}_{\vec{n},1} \cdot \mathbf{S}_{\vec{n},2} + J' \ldots$

dimer aspect of
$S = 1$ (Haldane) chain:
valence bond picture:

dimer aspect of
two leg ladder:

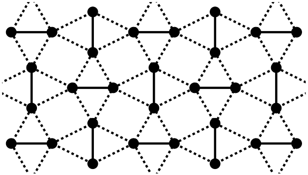


lowest excited state is triplet $\approx$ excited dimer

dimers interacting in 2D / 3D

orthogonal dimers in 2D

($SrCu_2(BO_3)_2$):

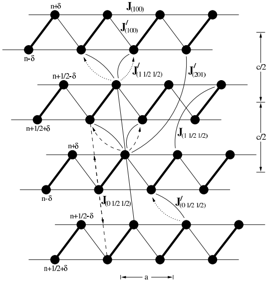


exact dimer ground state

magnetization plateaus

interacting dimer type ladders

($KCuCl_3$, $TlCuCl_3$, NH_4CuCl_3):



magnetization plateaus

triplet condensation:
'BEC of magnons'

zigzag ladder in a magnetic field

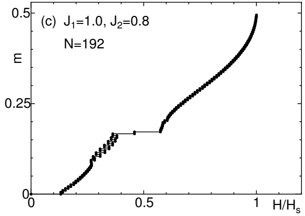
NN and NNN exchange: $\mathcal{H} = \sum_n \left(J\,\vec{S}_n \cdot \vec{S}_{n+1} + J_2\,\vec{S}_n \cdot \vec{S}_{n+2} - \mu\,H\,S_n^z\right)$

magnetization plateau exists:

$p(S - m)$ is integer:

$p = 3, S = 1/2, m = 1/6$

Okunishi and Tonegawa, PRB '03;
Yamanaka, Oshikawa, Affleck, PRL '97



zigzag ladder in a magnetic field

NN and NNN exchange:

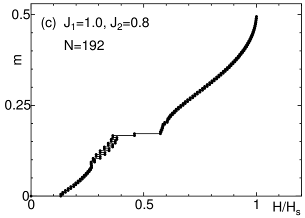
$$\mathcal{H} = \sum_n \left(J\, \vec{S}_n \cdot \vec{S}_{n+1} + J_2\, \vec{S}_n \cdot \vec{S}_{n+2} - \mu\, H\, S^z_n \right)$$

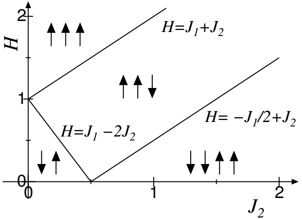
magnetization plateau exists:

$p(S - m)$ is integer:

$p = 3, S = 1/2, m = 1/6$

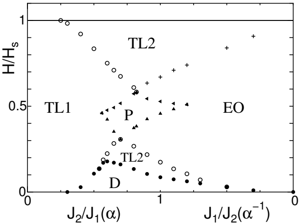
Okunishi and Tonegawa, PRB '03;
Yamanaka, Oshikawa, Affleck, PRL '97





Ising

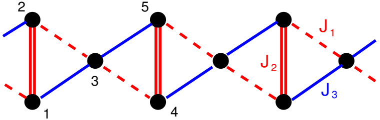
⟶



Heisenberg

$Cu_3(CO_3)_2(OH)_2$ = azurite: a real material

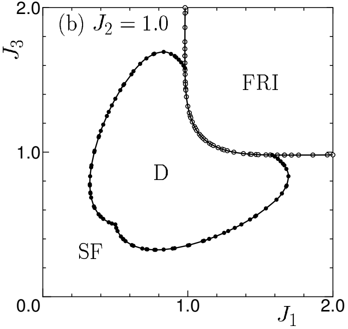
distorted diamond chain:



alternative view:

chain with NN interactions $\left(J_1\ J_2\ J_1\right)$

and NNN interactions $\left(J_3\ 0\ J_3\right)$



prototypes of low dimensional magnets:

- S=1/2 Heisenberg chain
- 1D quantum spin systems with gap and rotationally invariant exchange
- $S = 1/2$ chain with orbital degree of freedom
- 2D S=1/2 Heisenberg magnets

low D prototypes (3): $S = 1/2$ chain with orbital degree of freedom

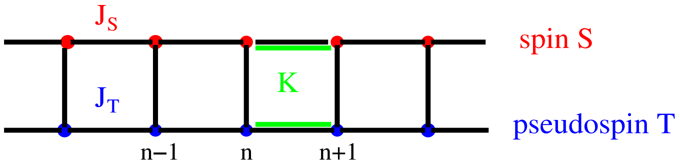
$$\mathcal{H} = J_S \sum_n (\mathbf{S_n} \cdot \mathbf{S_{n+1}}) + J_T \sum_n (\mathbf{T_n} \cdot \mathbf{T_{n+1}})$$
$$+ K \sum_n (\mathbf{S_n} \cdot \mathbf{S_{n+1}})(\mathbf{T_n} \cdot \mathbf{T_{n+1}})$$

Kugel-Khomskii model:

electron with two orbital states at each site, S=1/2, T=1/2

$\equiv$ spin ladder with leg-leg biquadratic interaction

$$\left\{ U(\text{same orbital}), U'(\text{different orbitals}), J(\text{Hund}) \right\} \cong \left\{ J_S, J_T, K \right\}$$

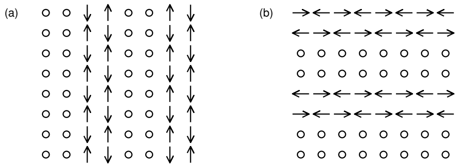


prototypes of low dimensional magnets:

- S=1/2 Heisenberg chain
- 1D quantum spin systems with gap and rotationally invariant exchange
- $S = 1/2$ chain with orbital degree of freedom
- 2D S=1/2 Heisenberg magnets

low D prototypes (4): 2D $S = 1/2$ Heisenberg antiferromagnets

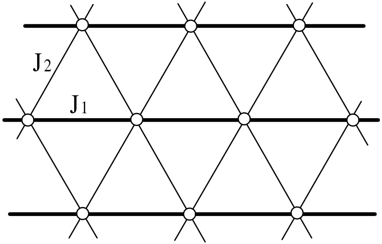
- square lattice with NN exchange (La_2CuO_4):
  Néel phase with long range order and spin waves
  four spin (ring) exchange at higher energies
- doped square lattice ($La_{2-x}Sr_xCuO_4$, HTSC):
  charges induce stripes / ladder character



J.S. Tranquada, '05

2D lattices with frustration:

- square lattice with NN and NNN exchange:
  satisfies purist's view of a spin liquid phase: no dimer aspects
- triangular lattice with spatial anisotropy (Cs_2CuCl_4):
  spin liquid / 2D spinons (?)



Coldea, Tennant, Tylczynski '00, '03 ...

quantum phases in localized spin systems

quantum phases differ in

groundstates:	ferro-, antiferro, chiral order
	disorder: dimers, spin liquid
	Heisenberg, XY, Ising symmetry
excitations	spinons (the quarks of Solid State Physics)
	excitation gaps for isotropic interactions
	quantum solitons

tune through quantum phase diagrams by varying

materials, doping, pressure and magnetic field

realize external magnetic fields

high magnetic field: $\mu H \approx \mathcal{O}(J) \approx \mathcal{O}(\mathrm{meV})$

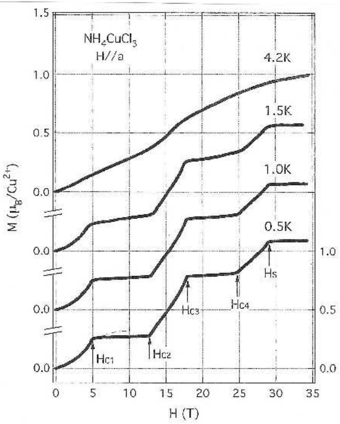
for $g = 2.2$: 1 meV $\approx$ 8 T, 1 K $\approx$ 0.7 T

specific heat, magnetization:
up to 90 T (Tokyo)

NMR, ESR:
up to 35 T (Grenoble)

x-ray-, neutron scattering:
up to 15 T (Berlin)

example:
magnetization plateaus
in NH_4CuCl_3

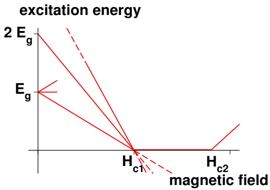


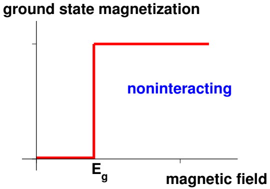
Shiramura, Tanaka '97

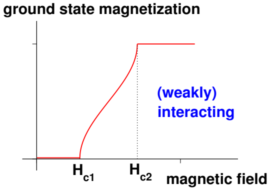
dimer condensation in magnetic field

$$\mathcal{H} = J \sum_{\vec{n}} \mathbf{S}_{\vec{n},1} \cdot \mathbf{S}_{\vec{n},2} + \sum_{\vec{n},\vec{n}'} J' \ldots - H \sum_{\vec{n},\alpha} S^z_{\vec{n},\alpha}$$

triplet excitation gap
closes at $H = H_{c1} = E_g$
saturation at H_{c2}



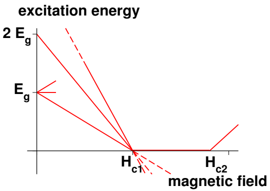




Magnetic field: Haldane meets Luttinger

when lowest Zeeman triplet condenses:
truncate Hilbert space

$4^L \rightarrow 2^L$



each ladder rung is either singlet or $S^z = 1$: map to

fermions or hard core bosons or S=1/2

effective S=1/2 chain for two leg ladder is:

$$J^{xy}_{\rm eff} = J_{\rm leg}, \quad J^z_{\rm eff} = \frac{1}{2} J_{\rm leg}, \quad H_{\rm eff} = H - \frac{1}{2} J_{\rm leg} - J_{\rm rung}$$

$J^z_{\rm eff} < J^{xy}_{\rm eff}$: intermediate phase is Luttinger liquid

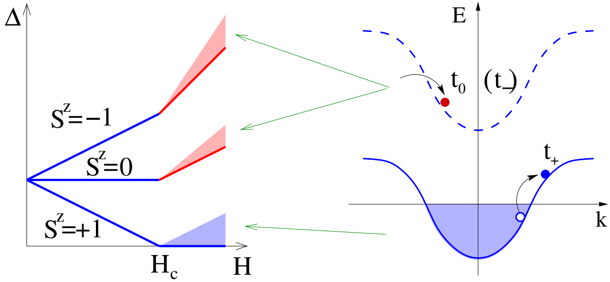
Mila '00 / Tsvelik and Giamarchi '03

NDMAP

isotropic 1D system: critical phase

ground state band: map to fermions

higher bands: “mobile impurities”



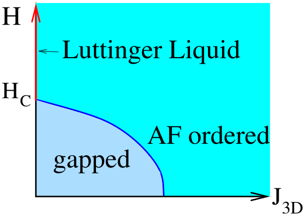
response: edge singularities, i.e. $S(q=\pi,\omega) \propto (\omega - \Delta_\mu)^{-\alpha}$

Fermi sea rearrangement $\Rightarrow$ change of slope at $H = H_c$

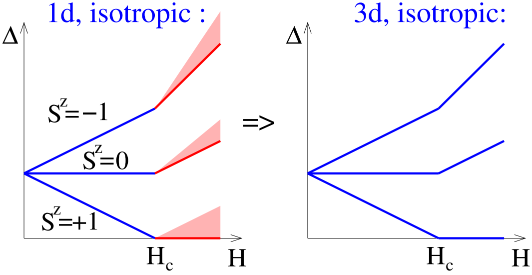
A.Furusaki & S.-C.Zhang ’99 / A. Kolezhuk & HJM ’02

3D interactions: critical phase becomes unstable

- U(1) *spontaneously* broken
  field-induced AF order above H_c
- continua collapse
  quasiparticle response



"Bose-Einstein condensation of magnons"



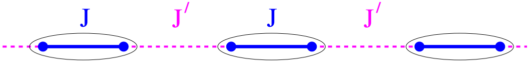
T.Nikuni et al. '00 / Ch.Rüegg et al. '03

NDMAP

dimer field theory

Idea:

- take a generic weakly coupled system: $S = \frac{1}{2}$ dimer chain

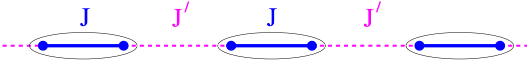


- introduce anisotropy: $J_x S_1^x S_2^x + J_y S_1^y S_2^y + J_z S_1^z S_2^z$
- carry the results over to strongly coupled systems (e.g. $S = 1$ Haldane chain)

dimer field theory

Idea:

- take a generic weakly coupled system: $S = \frac{1}{2}$ dimer chain



- introduce anisotropy: $J_x S_1^x S_2^x + J_y S_1^y S_2^y + J_z S_1^z S_2^z$
- carry the results over to strongly coupled systems (e.g. $S = 1$ Haldane chain)

Tool: dimer coherent states

(A. Kolezhuk '96)

$$|\Psi_{\text{dimer}}\rangle = \sqrt{1 - A^2 - B^2}|s\rangle + (\vec{A} + i\vec{B}) \cdot |\vec{t}\rangle$$

$$\langle \vec{S}_1 + \vec{S}_2 \rangle = 2(\vec{A} \times \vec{B}) \quad \mapsto \text{magnetization}$$

$$\langle \vec{S}_1 - \vec{S}_2 \rangle = 2\vec{A}\sqrt{1 - A^2 - B^2} \quad \mapsto \text{staggered magnetization}$$

$$\langle \vec{S}_1 \times \vec{S}_2 \rangle = \vec{B}\sqrt{1 - A^2 - B^2} \quad \mapsto \text{vector chirality}$$

dimer field theory Lagrangean:

- φ^4-type theory for a **complex** bosonic field ($A, B \ll 1$) with
- **generally, two sets of "stiffness constants":** $m_i \neq \widetilde{m}_i$

$$\widetilde{m}_x = \frac{1}{2}(J_y + J_z) \text{ etc.,} \qquad m_i = \widetilde{m}_i - J'$$

$$\lambda = J', \quad \lambda_1 = 2J', \quad \lambda_2 = -J'$$

NDMAP

dimer field theory Lagrangean:

- φ^4-type theory for a **complex** bosonic field ($A, B \ll 1$) with
- **generally, two sets of "stiffness constants":** $m_i \neq \widetilde{m}_i$

$$\widetilde{m}_x = \frac{1}{2}(J_y + J_z) \text{ etc.,} \qquad m_i = \widetilde{m}_i - J'$$

$$\lambda = J', \quad \lambda_1 = 2J', \quad \lambda_2 = -J'$$

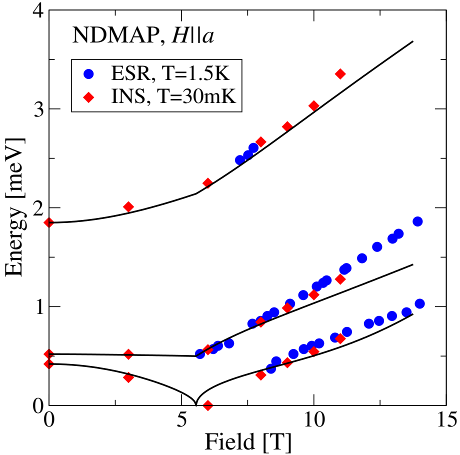
NDMAP

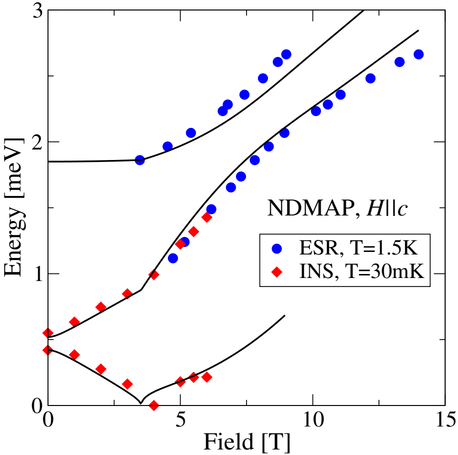
- effective Lagrangean after integrating out B:

$$\mathcal{L}_{\text{eff}} = \frac{1}{\widetilde{m}_i}\left\{\hbar^2(\partial_t A_i)^2 - V_i^2(\partial_x A_i)^2\right\} - 2\frac{\hbar}{\widetilde{m}_i}(\vec{H} \times \vec{A})_i \partial_t A_i - U_2 - U_4$$

- **anisotropic Zeeman term** (due to $\widetilde{m}_i$)
- **anisotropic interaction** at $H \neq 0$ (due to λ_1, λ_2)

Application: $Ni(C_5H_{14}N_2)_2N_3(PF_6)$ (NDMAP)



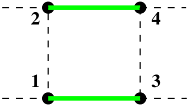


Neutrons: Zheludev et al.'03, '04 ESR: Hagiwara et al. '03

▸ $TlCuCl_3$ ◂ back

add four spin interactions

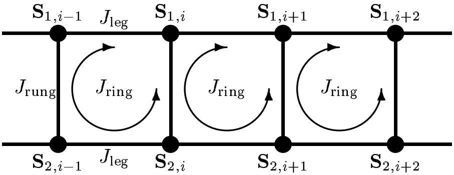
basic building block
of ladders and cuprates
is plaquette with four spins



general plaquette hamiltonian has 9 parameters
(+ arbitrary constant):

6 two spin interactions: 2 rungs, 2 legs, 2 diagonals
3 four spin interactions: leg-leg, diagonal-diagonal, rung-rung

exact ground states exist for some parameter sets a particular linear combination amounts to cyclic (ring) exchange:



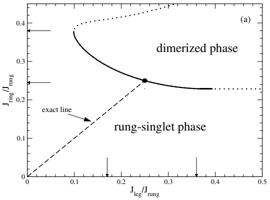
$$\mathcal{H} = \sum_{\text{plaquettes}} \frac{1}{2}J(P_{12}+P_{34}) + \frac{1}{2}J_{\text{leg}}(P_{13}+P_{24}) + \frac{1}{2}J_{\text{ring}}(P_{1243}+P_{1243}^{-1})$$

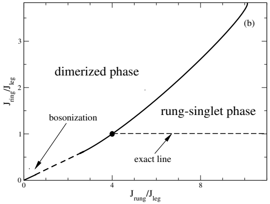
$$= \sum_{\text{plaquettes}} (J + J_{\text{ring}})(\vec{S}_1\cdot\vec{S}_2 + \vec{S}_3\cdot\vec{S}_4)$$

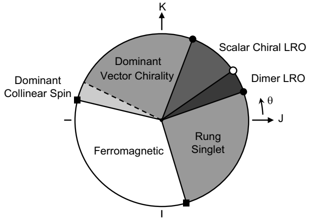
$$+(J_{\text{leg}} + \frac{1}{2}J_{\text{ring}})(\vec{S}_1\cdot\vec{S}_3 + \vec{S}_2\cdot\vec{S}_4) + \frac{1}{2}J_{\text{ring}}(\vec{S}_1\cdot\vec{S}_4 + \vec{S}_2\cdot\vec{S}_3)$$

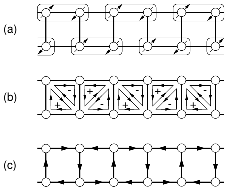
$$+2J_{\text{ring}}\{(\vec{S}_1\cdot\vec{S}_2)(\vec{S}_3\cdot\vec{S}_4) + (\vec{S}_1\cdot\vec{S}_3)(\vec{S}_2\cdot\vec{S}_4) - (\vec{S}_1\cdot\vec{S}_4)(\vec{S}_2\cdot\vec{S}_3)\}$$

phase diagram including ring exchange









staggered dimer

scalar chiral

vector chiral

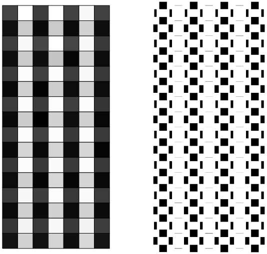
Müller, Vekua and HJM ’02 / Läuchli, Schmid and Troyer ’03

ring exchange in 2D

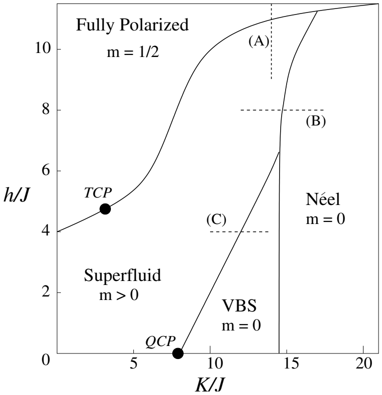
La_2CuO_4: ring exchange required to describe the dispersion of zone boundary magnons

S=1/2 model with ring exchange:

ring exchange introduces new quantum phases



Sandvik et al. '02 / '04



xy=superfluid phase • striped = VBS phase • Neel phase

from Ising domain walls to isotropic spinons

start from Ising limit $\mathcal{H} = \frac{1}{2} J \sum \sigma_n^z \sigma_{n+1}^z$

ground state ↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓ . . .

excited states:

domain wall ↑ ↓ ↑ ↓ ¦ ↓ ↑ ↓ ↑ ↓ ↑ . . .

Δ E = |J|

spin clusters ↑ ↓ ↑ ↓ ¦ ↓ ¦ ↓ ↑ ↓ ↑ ↓ . . .

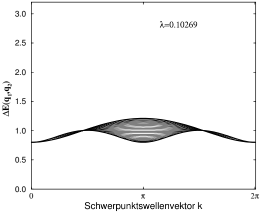
Δ E=2|J| ↑ ↓ ↑ ↓ ¦ ↓ ↑ ↓ ¦ ↓ ↑ ↓ . . .

domain wall = soliton
→ spinon

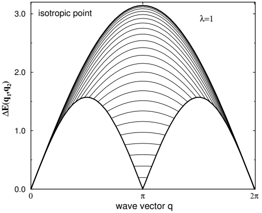
- spin cluster = 'spin wave'
  = two spinon continuum

spinon continua in S=1/2 chains

$\approx$ Ising limit

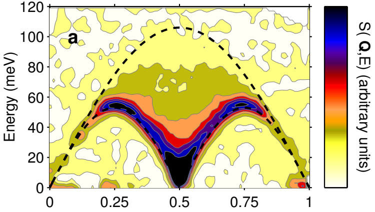


Heisenberg limit



$KCuF_3$, S=1/2:

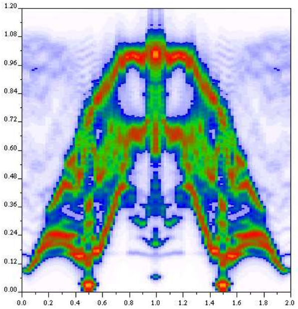
B. Lake, A. Tennant, Nature Materials 2004

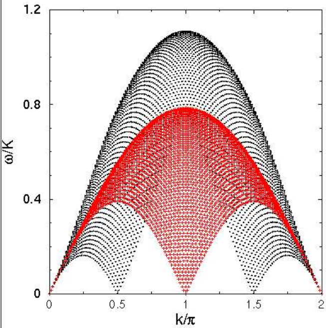


3 sets of spinons at the SU(4) point

SU(4) symmetric Kugel-Khomskii model: $J_S = J_T = K/4$ in

$$\mathcal{H} = J_S \sum_n (\mathbf{S_n} \cdot \mathbf{S_{n+1}}) + J_T \sum_n (\mathbf{T_n} \cdot \mathbf{T_{n+1}})$$
$$+K \sum_n (\mathbf{S_n} \cdot \mathbf{S_{n+1}})(\mathbf{T_n} \cdot \mathbf{T_{n+1}})$$





Affleck '89, Schollwöck '00

Summary

Low D quantum magnets with general exchange interactions cover a vast number of materials and theoretical models.

- an external magnetic field is the ideal accessible parameter to tune materials through different quantum phases

Summary

Low D quantum magnets with general exchange interactions cover a vast number of materials and theoretical models.

- an external magnetic field is the ideal accessible parameter to tune materials through different quantum phases
- multispin exchange interactions such as
  - cyclic = ring exchange
  - interactions between spin and orbital degrees of freedom
  - phonon induced exchange between pairs of spins

  give rise to new quantum phases and quantum phase transitions

Summary

Low D quantum magnets with general exchange interactions cover a vast number of materials and theoretical models.

- an external magnetic field is the ideal accessible parameter to tune materials through different quantum phases
- multispin exchange interactions such as
  - cyclic = ring exchange
  - interactions between spin and orbital degrees of freedom
  - phonon induced exchange between pairs of spins

  give rise to new quantum phases and quantum phase transitions
- excitation continua result from the fractionalization of spin

Summary

Low D quantum magnets with general exchange interactions cover a vast number of materials and theoretical models.

- an external magnetic field is the ideal accessible parameter to tune materials through different quantum phases
- multispin exchange interactions such as
  - cyclic = ring exchange
  - interactions between spin and orbital degrees of freedom
  - phonon induced exchange between pairs of spins

  give rise to new quantum phases and quantum phase transitions
- excitation continua result from the fractionalization of spin

Challenges:

- Experiment: new materials / high magnetic fields
- Theory: 2D / symmetries and quantum phases

Thanks to

A.K. Kolezhuk, Kiev / ITP Hannover / Harvard

PhD work in Hannover:

S. Brehmer, U. Neugebauer, M. Müller, T. Vekua

cooperations / coauthors:

U. Schollwöck, TH Aachen

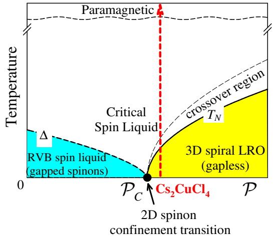
N. Nagaosa, S. Uchida, U Tokyo

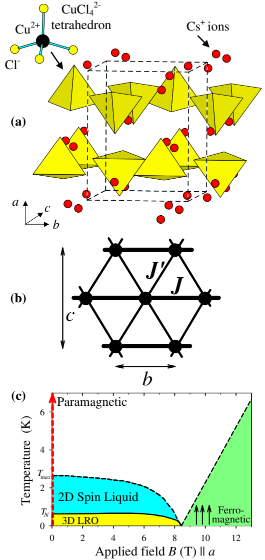
H. Tanaka, A. Oosawa, TIT
M. Matsuda, K. Katsumata, H. Hagiwara, Z. Honda, RIKEN
A. Zheludev, Oak Ridge

HJM and AK Kolezhuk, One-dimensional Magnetism
review article in: Quantum Magnetism, Lect. Notes Phys. **645**, 1 (2004)

Cs_2CuCl_4 phase diagrams

back





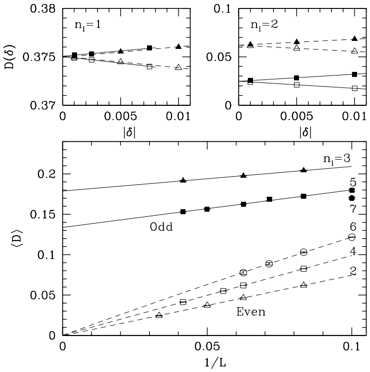
purists' view of a spin liquid

spin liquid requires:

no spontaneous dimerization
(Majumdar-Ghosh disqualifies)

only one electron per unit cell
(ladder and S=1 chain disqualify)

purists' example is (Capriotti):
even leg ladder / 2D HAF
with NN and NNN exchange:
gs not dimerized



dimer susceptibility

back

L. Capriotti, D.J. Scalapino and S.R. White, PRL '04

dimer field theory Lagrangean:

- a φ^4-type theory for a **complex** bosonic field ($A, B \ll 1$)

$$\mathcal{L} = \hbar\left(\vec{A}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{B}\cdot\frac{\partial\vec{A}}{\partial t}\right) - \frac{1}{2}J'\left(\frac{\partial\vec{A}}{\partial x}\right)^2 - m_i\, A_i^2 - \widetilde{m}_i\, B_i^2$$
$$+ \; 2\vec{H}\cdot(\vec{A}\times\vec{B}) - \lambda\, A^4 - \lambda_1\,(A^2B^2) - \lambda_2\,(\vec{A}\cdot\vec{B})^2$$

- **generally, two sets of "stiffness constants":** $m_i \neq \widetilde{m}_i$

dimer field theory Lagrangean:

- a φ^4-type theory for a **complex** bosonic field ($A, B \ll 1$)

$$
\begin{aligned}
\mathcal{L} &= \hbar\left(\vec{A}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{B}\cdot\frac{\partial\vec{A}}{\partial t}\right) - \frac{1}{2}J'\left(\frac{\partial\vec{A}}{\partial x}\right)^2 - m_i\, A_i^2 - \widetilde{m}_i\, B_i^2 \\
&+ 2\vec{H}\cdot(\vec{A}\times\vec{B}) - \lambda\, A^4 - \lambda_1\,(A^2B^2) - \lambda_2\,(\vec{A}\cdot\vec{B})^2
\end{aligned}
$$

- **generally, two sets of "stiffness constants":** $m_i \neq \widetilde{m}_i$

$$\widetilde{m}_x = \tfrac{1}{2}(J_y + J_z)\ \text{etc.}, \quad m_i = \widetilde{m}_i - J'$$

$$\lambda = J', \quad \lambda_1 = 2J', \quad \lambda_2 = -J'$$

effective Lagrangean for real field $\vec{A}$:

back

$$\vec{B} = \widehat{Q}\vec{F}, \quad \vec{F} = -\hbar\frac{\partial\vec{A}}{\partial t} + (\vec{H}\times\vec{A}), \quad Q_{ij} = \frac{\delta_{ij}}{\widetilde{m}_i} - \lambda_1\frac{\delta_{ij}\vec{A}^2}{\widetilde{m}_i^2} - \lambda_2\frac{A_iA_j}{\widetilde{m}_i\widetilde{m}_j}$$

$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{\widetilde{m}_i}\left\{\hbar^2\left(\frac{\partial A_i}{\partial t}\right)^2 - V_i^2\left(\frac{\partial A_i}{\partial x}\right)^2\right\} - 2\frac{\hbar}{\widetilde{m}_i}(\vec{H}\times\vec{A})_i\frac{\partial A_i}{\partial t} - U_2 - U_4$$

here
$$U_2(\vec{A}) = m_i\,A_i^2 - \frac{1}{\widetilde{m}_i}(\vec{H}\times\vec{A})_i^2$$

$$U_4(\vec{A},\frac{\partial\vec{A}}{\partial t}) = \lambda\,A^4 + \lambda_1\,A^2\frac{1}{\widetilde{m}_i^2}F_i^2 + \lambda_2\,\frac{A_iA_j}{\widetilde{m}_i\widetilde{m}_j}F_iF_j$$

Some properties of the model:

- zero-field gaps: $\Delta_z = \sqrt{m_z\ \widetilde{m}_z}$ etc.
- critical fields: $H_c^{(z)} = \min\left\{ \sqrt{m_x\ \widetilde{m}_y}\ ,\ \sqrt{\widetilde{m}_x\ m_y} \right\}$ etc.
- reduces to known theories in limiting special cases:

$$\widetilde{m}_i = \widetilde{m} \quad \Rightarrow \quad \text{Affleck} \quad H_c^{(z)} = \Delta_z$$

$$\widetilde{m}_i = m_i = \Delta_i \quad \Rightarrow \quad \text{Mitra\&Halperin} \quad H_c^{(z)} = \sqrt{\Delta_x \Delta_y}$$

- **allows to avoid the OP direction problem**

▸ . . .

How many fitting parameters?

nine constants present in the theory:

$$\lambda, \quad \lambda_{1,2}, \quad m_{x,y,z}, \quad \widetilde{m}_{x,y,z}$$

but only three are left if we fix critical fields and zero-field gaps:

- six equations

$$H_{c,z}^2 = m_x \widetilde{m}_y, \qquad H_{c,x}^2 = m_y \widetilde{m}_z, \qquad H_{c,y}^2 = m_x \widetilde{m}_z,$$
$$\Delta_z^2 = m_z \widetilde{m}_z, \qquad \Delta_x^2 = m_x \widetilde{m}_x, \qquad \Delta_y^2 = m_y \widetilde{m}_y$$

  define five independent constraints, so from ($m_{x,y,z}$, $\widetilde{m}_{x,y,z}$) only one parameter is free (overall scale)
- mode energies depend only on the ratios $\lambda_1/\lambda, \quad \lambda_2/\lambda$

The OP direction at $H > H_c$

back

- in Mitra&Halperin, for $\vec{H} \parallel z$: **inherent problem**

$$U_2 = (\Delta_x - \frac{H^2}{\Delta_y})\, A_x^2 + (\Delta_y - \frac{H^2}{\Delta_x})\, A_y^2 + \Delta_z\, A_z^2,$$

if $\Delta_x < \Delta_y$ (x = easy axis) then $\vec{A} \parallel \vec{y}$ for $H > H_c$

i.e. the staggered order *always along the harder axis!*

The OP direction at $H > H_c$

back

- in Mitra&Halperin, for $\vec{H} \parallel z$: **inherent problem**

$$U_2 = (\Delta_x - \frac{H^2}{\Delta_y})\, A_x^2 + (\Delta_y - \frac{H^2}{\Delta_x})\, A_y^2 + \Delta_z\, A_z^2,$$

  if $\Delta_x < \Delta_y$ (x = easy axis) then $\vec{A} \parallel \vec{y}$ for $H > H_c$

  i.e. the staggered order *always along the harder axis!*

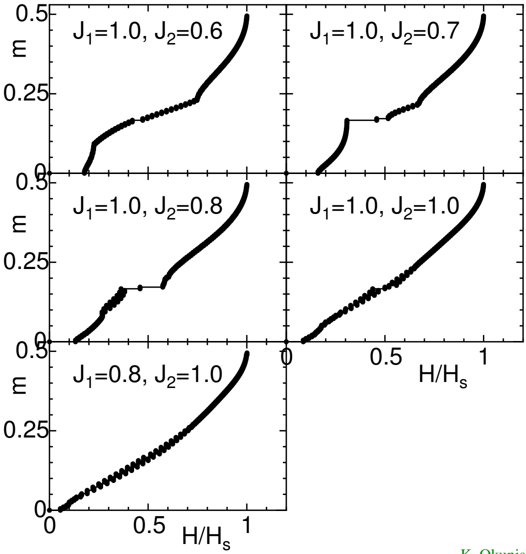
- **not the case for the proposed theory:**

$$U_2 = (m_x - \frac{H^2}{\widetilde{m}_y})\, A_x^2 + (m_y - \frac{H^2}{\widetilde{m}_x})\, A_y^2 + m_z\, A_z^2,$$

if $\quad m_x/\widetilde{m}_x < m_y/\widetilde{m}_y \quad$ then $\quad \vec{A} \parallel \vec{x}, \quad H_c = \sqrt{m_x \widetilde{m}_y}$

if $\quad m_x/\widetilde{m}_x > m_y/\widetilde{m}_y \quad$ then $\quad \vec{A} \parallel \vec{y}, \quad H_c = \sqrt{m_y \widetilde{m}_x}$

magnetization plateau in the NN-NNN chain

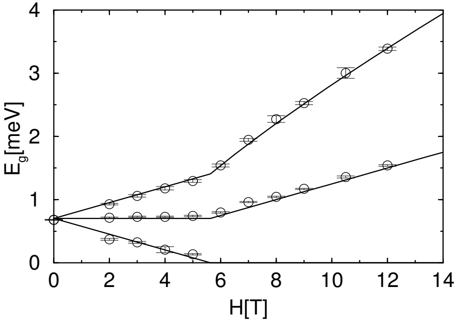


K. Okunishi and T. Tonegawa, JPSJ 72, 479 (2003)

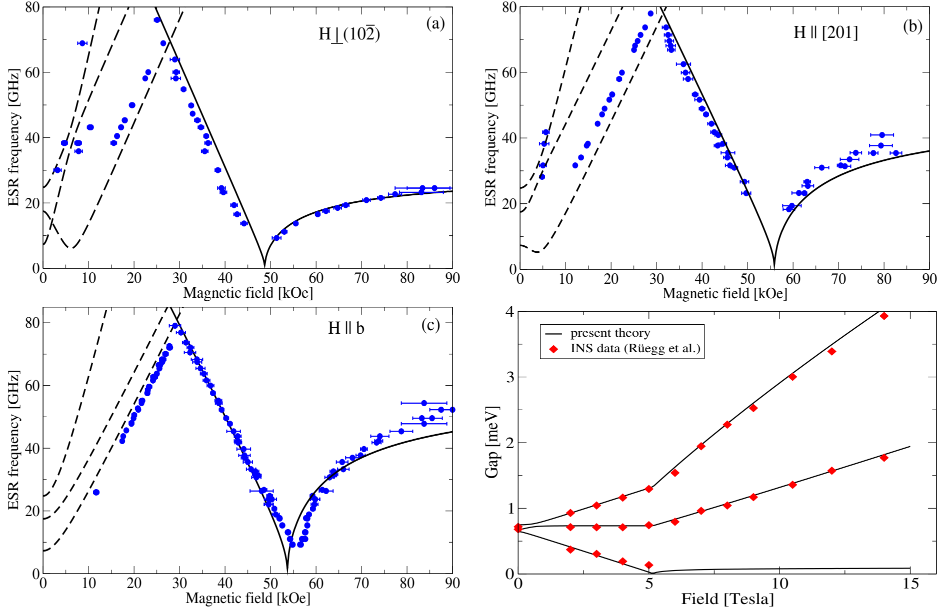
◂ back

Application: $TlCuCl_3$ (3D-coupled $S = \frac{1}{2}$ dimers)

- INS (Rüegg et al '03): the lowest mode is gapless ("BEC")?



- ESR Glazkov et al. '03: gap reopens at $H > H_c \Rightarrow$ anisotropy!
- consistent with exchange anisotropy $< 1\%$ (intra- and inter-dimer)



AK, V.Glazkov, H.Tanaka and A.Oosawa, PRB'04

back